

# Modeling with Possible Non-Ignorable Dropout in Longitudinal Studies

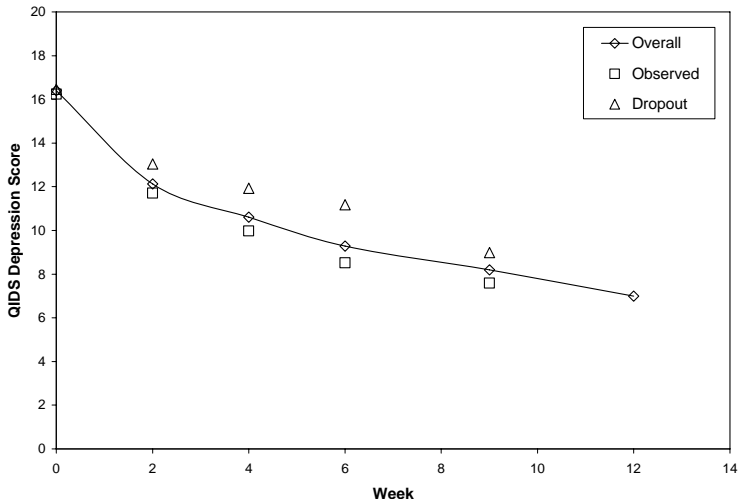
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- Overview of key missing data models presented in the general latent variable framework of Mplus
- Current approaches: Shared set of latent variables for the outcome and missing data processes
- New approach: Disentangling the outcome process and the missing data process

# Example: Longitudinal Data From An Antidepressant Trial (STAR\*D) $n = 4041$

Subjects treated with citalopram (Level 1). No placebo group



- $y$ : outcome vector - but  $y$  is not the only data
- $m$ : missing data indicator vector

- MCAR

$$P(m_i | y_i^{obs}, y_i^{mis}) = P(m_i) \quad (1)$$

- MAR

$$P(m_i | y_i^{obs}, y_i^{mis}) = P(m_i | y_i^{obs}) \quad (2)$$

$$ML [y, m] = ML [y] \quad (3)$$

- NMAR (non-ignorable or informative missingness):  
None of the above. Missingness predicted by latent variables

# NMAR Modeling: Joint Analysis Of The Outcome And Missing Data Processes

- $y$ : outcome vector
- $m$ : missing data indicator vector
  - Focus has typically been on dropout with  $m$  replaced by dropout indicators  $d$ . Often,  $d$ 's are scored as discrete-time survival (event history) indicators. For example, dropout after 3rd visit: 0 0 0 1 999 999. But  $d$ 's can also be dropout dummy variables.
  - Mplus creates  $m$  and  $d$  using Data Missing
- $c$ : latent class variable

$$[y, m] = [y] [m|y], \textit{ Selection modeling} \quad (4)$$

$$= [m] [y|m], \textit{ Pattern - mixture modeling} \quad (5)$$

$$= \sum_c [c] [m|c] [y|c] \textit{ Shared - parameter modeling} \quad (6)$$

- Each NMAR model involves untestable assumptions - comparing results from several models gives a sensitivity analysis
- Overviews in Albert and Follman (2008), Little (2008)

# The unifying theme of latent variables

## Continuous Latent Variables

- Factors
- Random effects
- Frailties, liabilities
- Variance components
- Missing data
- Bayesian parameter priors

## Categorical Latent Variables

- Latent classes
- Clusters
- Finite mixtures
- Missing data

- Mplus support from National Institutes of Health Small Business Innovation Research contracts and grants

Several programs in one, fully integrated:

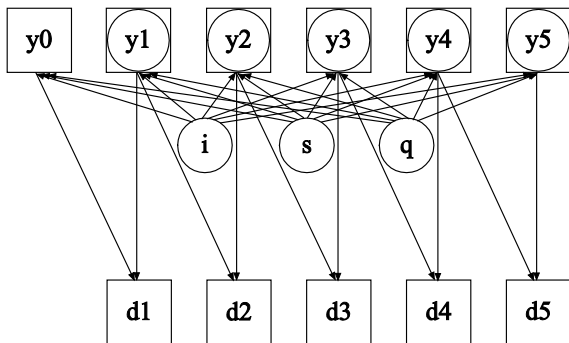
- Exploratory factor analysis
- Structural equation modeling
- Item response theory analysis
- Latent class analysis
- Latent transition analysis (Hidden Markov modeling)
- Survival analysis
- Growth modeling
- Multilevel analysis
- Complex survey data analysis
- Bayesian analysis using MCMC
- Monte Carlo simulation

- A large set of NMAR models can be estimated in the general latent variable modeling framework of Mplus ([www.statmodel.com](http://www.statmodel.com))
- Maximum-likelihood estimation using EM in combination with FS and QN
- Number of classes informed by BIC and bootstrapped likelihood-ratio test
- Bayesian MCMC analysis available as well using DIC and PPC



# Diggle-Kenward Selection Model

(d's: dropout indicators in line with discrete-time survival)

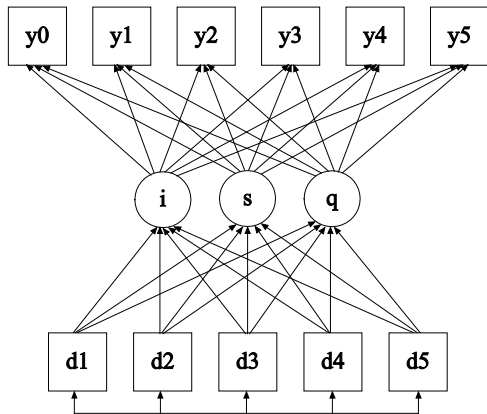


Diggle-Kenward (1994) in Applied Statistics

Logistic regression of  $d_t$  on observed  $y_{t-1}$  and observed/unobserved  $y_t$

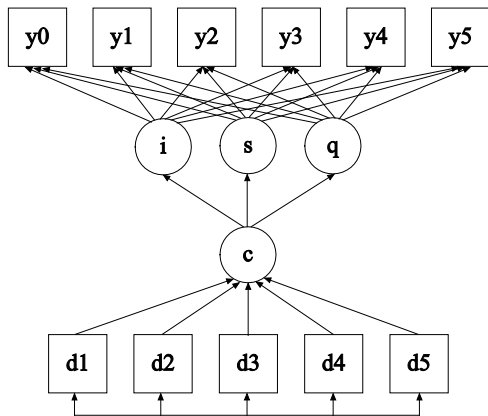
# Pattern-Mixture Model

(d's: dropout pattern dummies)



Hedeker-Gibbons (1997) in *Psychological Methods*,  
Demirtas-Schafer (2003) in *Statistics in Medicine*

# Roy-Dantan Latent Class Dropout Model (d's: dropout pattern dummies)



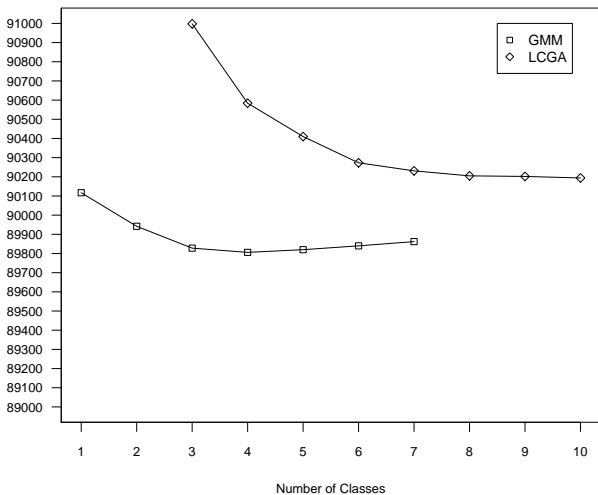
Roy (2003) Biometrics:  $c$  on dropout time

Dantan et al (2008) Int'l J. of Biostat:  $c$  on dropout dummies

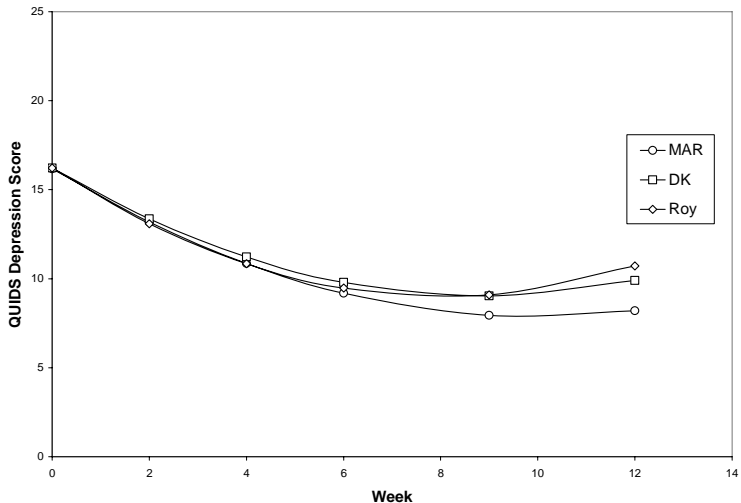
# Summary Of Mixture Modeling Of STAR\*D Data Using Dropout Pattern Dummies

Model	Loglikelihood	#par.s	BIC
Pattern-mixture	-44946	27	90117
Roy 2c	-44871	24	89942
Roy 3c	-44777	33	89828
<b>Roy 4c</b>	<b>-44728</b>	<b>42</b>	<b>89806</b>
Roy 5c	-44698	51	89820

# An aside: BIC curves for Roy-GMM and Roy-LCGA



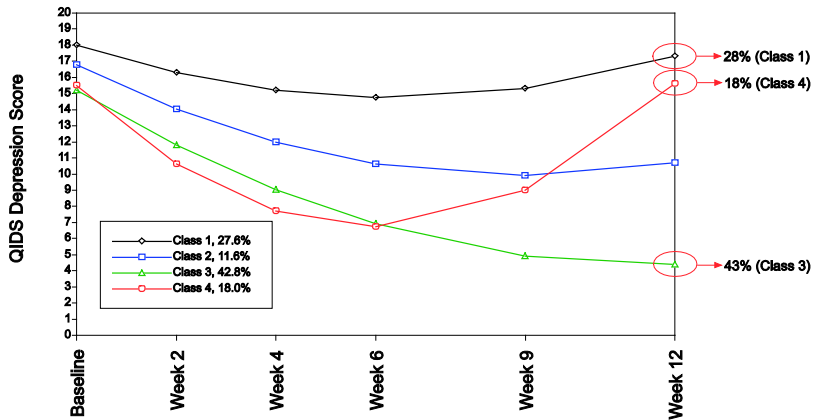
# Depression Mean Curves Estimated Under MAR, Diggle-Kenward NMAR, And Roy NMAR



# Disadvantages Of Pattern-Mixture And Roy Latent Class Dropout Modeling

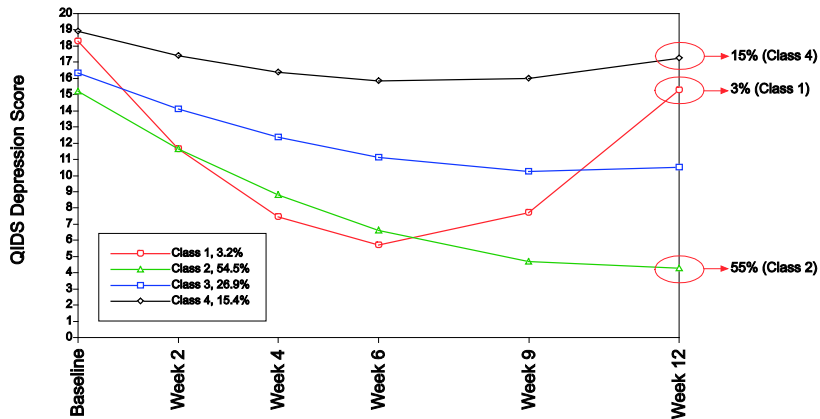
- For both approaches the intention is to mix the parameter estimates over the patterns/classes to obtain an overall estimated growth curve
  - This mixing may hide substantively interesting trajectory classes

# 4-Class Roy-Dantan Latent Class Dropout Model





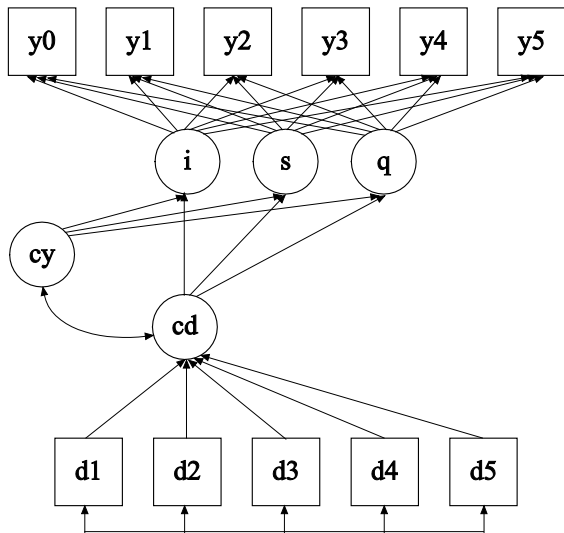
# 4-Class MAR Model (Y Outcomes Only)



# Disadvantages Of Pattern-Mixture And Roy Latent Class Dropout Modeling

- Roy latent class dropout modeling forms classes based not only on the relationship between dropout and outcomes, but also based on the development of the outcomes over time
  - This may confound dropout classes with trajectory classes

# Muthén-Roy Model Using Two Latent Class Variables



# Model Comparisons

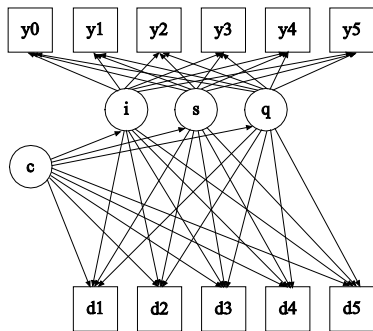
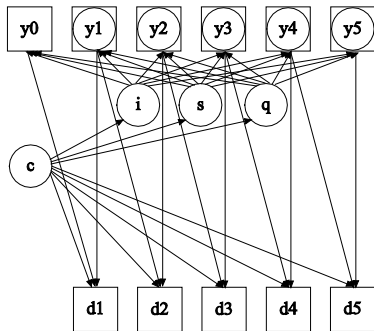
Model	Loglikelihood	#par.s	BIC
Pattern-mixture	-44946	27	90117
Roy 4c	-44728	42	89806
<b>Muthen 4c-Roy 2c</b>	<b>-44662</b>	<b>44</b>	<b>89689</b>

# Comparing Trajectory Class Percentages Across Models

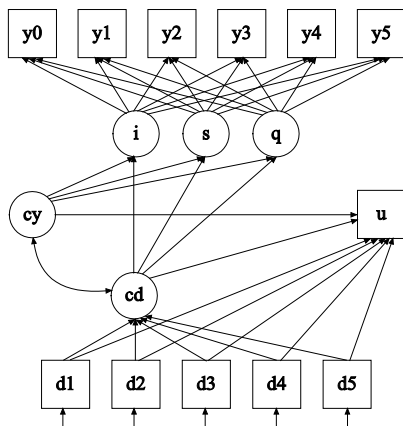
Adding dropout information gives a less favorable conclusion regarding drug response than assuming MAR

Model	Response class	Temporary response class	Non-response class
MAR 4 classes	55 %	3 %	15 %
NMAR models:			
Roy 4 classes	43 %	18 %	28 %
Muthen 4c-Roy 2c	32 %	19 %	32 %

# Selection models with latent classes: Generalized Diggle-Kenward and Beunckens et al.



# Muthén-Roy model extended to include an ultimate outcome



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